THE GRIPHAN ONLINE SCHOOL

UNIVERSITY LIBRARY

Mathematics for University Study

Algebra, functions, calculus and the habits of mathematical thinking

Nji Griphan

griphan.48hourrfprescue.com

Mathematics for University Study

Algebra, functions, calculus and the habits of mathematical thinking

Nji Griphan

The Griphan Online School — University Library

IMPRINT

This book is published for the Library of the Griphan Online School. Author and publisher of record: **Nji Griphan**. School address on the web: **griphan.48hourrfprescue.com**. Professional mailbox: **info@griphan.48hourrfprescue.com**. © 2026 Nji Griphan. This book was written for the students of this school. Students enrolled at the school may download and keep it for their own study. The library grows shelf by shelf until it carries the whole university.

A note on this book

University courses in science, agriculture, economics and computing all assume a working mathematics: algebra handled fluently, functions understood as machines, and the first ideas of calculus. This book rebuilds those foundations carefully, with the emphasis on understanding that survives examinations — every method explained,

worked and practised.

Contents

Chapter 1 — Numbers, Powers and Roots

Chapter 2 — Algebra — The Language of Relationships

Chapter 3 — Equations — Linear and Quadratic

Chapter 4 — Functions and Graphs

Chapter 5 — Introducing Calculus — Rates and Totals

Chapter 6 — Mathematical Thinking and Proof

Appendix — Worked Examples and Practice

Glossary of Key Terms

Chapter 1

Numbers, Powers and Roots

Fluency begins with number itself: fractions, decimals, percentages, powers and roots, handled accurately and estimated sensibly. This chapter also teaches the habit that protects all later work — checking whether an answer is the right size.

Fractions and percentages

A fraction is a division not yet performed; a percentage is a fraction of a hundred. Moving freely among fraction, decimal and percentage forms is basic literacy: a 12.5 per cent discount is an eighth off, and seeing that instantly is worth more than computing it slowly.

Powers

A power repeats multiplication: 2^5 = 32. The laws of indices — multiply powers of the same base by adding exponents, divide by subtracting — are bookkeeping, not magic, and they govern everything from compound interest to the binary arithmetic of computers.

Roots and reciprocals

The square root undoes squaring; the reciprocal undoes multiplication. Negative and fractional powers extend the same bookkeeping: a^(−1) = 1/a, a^(1/2) = √a. Once the notation is seen as one consistent system, formulas stop being spells and become sentences.

Estimation and sanity checks

Before calculating exactly, estimate: round the numbers, get the size, then compute. An answer wildly off its estimate announces an error while it is still cheap. Scientists estimate constantly; students who skip it submit hectares measured in grams.

Key points

- Fraction, decimal and percentage are one fact in three clothes.
- Index laws are bookkeeping for repeated multiplication.
- Estimate first; compute second; compare always.

Exercises

1. Express 0.375 as a fraction and a percentage.
2. Simplify 2^7 × 2^3 ÷ 2^4.
3. Estimate 4,812 × 0.19, then compute exactly and compare.

Chapter 2

Algebra — The Language of Relationships

Algebra generalises arithmetic: letters stand for numbers not yet known or deliberately left general. This chapter builds fluency in simplifying, expanding, factorising and rearranging — the manipulations every science course assumes you can do without thinking.

Expressions and simplification

An expression is a recipe of operations on numbers and letters. Simplification collects like terms — 3x + 5x = 8x — and applies operations in the agreed order. Like terms only: 3x + 5y stays as it is, and pretending otherwise is the commonest algebra error in first-year scripts.

Expanding and factorising

Expanding multiplies out brackets: 3(x + 4) = 3x + 12. Factorising is the same road backwards: 3x + 12 = 3(x + 4). Fluency is travelling both directions without hesitation, because equations are solved by whichever direction the problem demands.

Rearranging formulas

To make a different letter the subject, undo operations in reverse order, doing the same to both sides. The formula for a mean, total = mean × count, rearranges to count = total ÷ mean. Every science rearranges formulas daily; treat each rearrangement as an equation and the method never changes.

Substitution and checking

Substituting values into a formula is arithmetic with a map. The professional habit is the back-check: substitute your answer into the original relationship and confirm both sides agree. Thirty seconds of checking outranks ten minutes of re-derivation.

Key points

- Collect like terms only.
- Expanding and factorising are one road, two directions.
- Rearrange by undoing operations equally on both sides.

Exercises

1. Simplify $4a + 3b - a + 2b$.
2. Expand $(x + 5)(x - 2)$ and factorise $x^2 + 3x - 10$.
3. From $A = \pi r^2$, make r the subject (r positive).

Chapter 3

Equations — Linear and Quadratic

An equation states that two expressions are equal and asks for the values that make it true. This chapter solves the two families that dominate applications — linear equations, with one crossing point, and quadratics, with their two, one or no solutions.

Linear equations

Solve by isolating the unknown: undo addition, then multiplication, keeping both sides balanced. Word problems become linear equations once quantities are named with letters — the naming, not the solving, is where word problems are won.

Simultaneous equations

Two unknowns need two equations. Eliminate one unknown by adding or subtracting multiples of the equations, or substitute one expression into the other. Applications abound: two crops, two constraints; two prices, two totals.

Quadratic equations

Quadratics, with their x^2 term, arise wherever areas, projectiles or products of relationships appear. Solve by factorising where possible, and by the formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ in general — a formula worth memorising exactly, sign by sign.

The discriminant

The expression $b^2 - 4ac$ under the root — the discriminant — foretells the solutions: positive gives two real roots, zero gives one repeated root, negative gives none in real numbers. Reading it before solving is the mathematician's version of estimation: know what kind of answer to expect.

Key points

- Name quantities with letters; word problems fall after that.
- Two unknowns require two equations.
- The discriminant foretells two, one or no real solutions.

Exercises

1. Solve $5x - 7 = 3x + 9$.
2. Solve simultaneously: $x + y = 10$, $2x - y = 5$.
3. Solve $x^2 - 5x + 6 = 0$ by factorising and by the formula.

Chapter 4

Functions and Graphs

A function is a machine: input a number, output exactly one number. This chapter teaches the language of functions, the reading of graphs, and the straight line — slope and intercept — that models more of the world than any other curve.

Notation and evaluation

f(x) names the output of function f at input x. f(3) means 'run the machine at 3'. Functions may be given by formula, table or graph; fluency is translating among the three without noticing the joins.

Reading graphs

A graph shows a relationship at a glance: where it rises, falls, peaks and crosses the axes. Intercepts are where the graph meets the axes; turning points are where rise becomes fall. Most scientific figures are graphs, and reading them is a core university skill in itself.

The straight line

y = mx + c: m is the slope — output change per unit input — and c the intercept, the output at input zero. Slope is the mathematical form of every 'rate' in science: growth per week, cost per hectare, marks per hour of study.

Proportionality and models

When output is proportional to input, the graph is a line through the origin and the model is one multiplication.

Recognising proportionality — and its absence — is the beginning of mathematical modelling, the art of choosing the simplest relationship the facts allow.

Key points

- A function gives exactly one output per input.
- In $y = mx + c$, m is rate of change and c is the starting value.
- Read graphs for intercepts, turning points and trend before detail.

Exercises

1. For $f(x) = 2x^2 - 3$, find $f(0)$, $f(2)$, $f(-1)$.
2. Find the slope and intercept of $y = 4x - 7$ and interpret both for a cost model.
3. Sketch $y = -x + 5$, marking both intercepts.

Chapter 5

Introducing Calculus — Rates and Totals

Calculus answers two questions: how fast is something changing (differentiation) and how much has accumulated (integration). This chapter introduces both ideas honestly at foundation level — enough to meet them in science courses without fear.

Slopes of curves

A curve's slope changes from point to point; the slope at a point is the slope of the tangent there. Differentiation computes it: for $y = x^n$, the derivative is nx^{n-1}. So $y = x^2$ has slope $2x$ — flat at the origin, steep far out, exactly as its picture promises.

What derivatives mean

The derivative of position is speed; of a population, its growth rate; of cost, the cost of one more unit. Whenever a science says 'rate of change', a derivative is standing behind the words, and optimisation — finding maxima and minima where the derivative is zero — stands behind half the recommendations in economics and agriculture.

Integration — accumulation

Integration reverses differentiation and also computes totals: area under a curve, total rainfall from a rate, total growth from daily gains. The symbol $\int$ and the constant of integration are the paperwork; the idea is addition refined to continuity.

The fundamental connection

That differentiation and integration undo each other — the fundamental theorem — is the deepest single fact in this book. Rates and totals are two views of one process, and calculus is the machinery for passing between them.

Key points

- The derivative is the slope at a point; for x^n it is nx^{n-1}.
- Integration accumulates totals and reverses differentiation.
- Rates and totals are two views of one process.

Exercises

1. Differentiate $y = x^3$, $y = 5x^2$, $y = 7x$.
2. Where is the maximum of $y = -x^2 + 6x$? (Set the derivative to zero.)
3. Integrate $\int 4x\,dx$ and $\int x^2\,dx$.

Chapter 6

Mathematical Thinking and Proof

The final chapter is about the habits, not the topics: how mathematicians convince themselves and others — checking special cases, seeking counterexamples, and building arguments that do not leak.

Conjecture and counterexample

A pattern spotted in examples is a conjecture, not a theorem. One counterexample kills a conjecture outright — 'all primes are odd' dies at the number 2 — while a thousand examples cannot prove one. Test conjectures at the awkward values: zero, one, negatives, equals.

Proof — showing it must be so

A proof is an argument from accepted facts to the claim, with no step resting on 'it looks true'. Direct proof walks forward from the givens; proof by contradiction assumes the opposite and derives an impossibility. Reading proofs slowly, pen in hand, is how the skill is acquired.

Mathematical writing

State what you will show, show it in ordered steps, and conclude plainly. Define every letter you use. A correct argument badly written earns less than it deserves in examinations and persuades nobody in research; clarity is a mathematical virtue, not a literary extra.

Carrying mathematics forward

Keep the habits of this book — estimate, check, test the awkward case, write clearly — and every quantitative course that follows becomes a variation on familiar themes. Mathematics at university is not a new country; it is this country, explored further.

Key points

- Examples suggest; only proof establishes.
- One counterexample refutes any conjecture.
- Define your letters; order your steps; conclude plainly.

Exercises

1. Find a counterexample to 'every even number is divisible by 4'.

2. Prove that the sum of two odd numbers is even.

3. Test the claim '$x^2 > x$ for all x' at three values and repair the claim.

Appendix — Worked Examples and Practice

From Chapter 1 — Numbers, Powers and Roots

Worked example: 4,812 × 0.19. Estimate: about 5,000 × 0.2 = 1,000. Exact: 4,812 × 19 = 91,428, so × 0.19 = 914.28. The exact answer sits sensibly near the estimate of 1,000. Had the calculator shown 91.428 or 9,142.8, the estimate would have caught the slipped decimal point instantly.

From Chapter 2 — Algebra — The Language of Relationships

Worked example: $(x + 5)(x - 2) = x^2 - 2x + 5x - 10 = x^2 + 3x - 10$. Backwards: $x^2 + 3x - 10$ needs two numbers multiplying to -10 and adding to $+3$: they are $+5$ and -2, so the factorisation is $(x + 5)(x - 2)$ — the expansion read in reverse, confirming both directions at once.

From Chapter 3 — Equations — Linear and Quadratic

Worked example: $x^2 - 5x + 6 = 0$. Factorising: two numbers multiplying to 6 and adding to -5 are -2 and -3, so $(x - 2)(x - 3) = 0$ and $x = 2$ or $x = 3$. By the formula: $a = 1$, $b = -5$, $c = 6$; discriminant $25 - 24 = 1$; $x = (5 \pm 1)/2 = 3$ or 2. Two methods, one answer — agreement is the check.

From Chapter 4 — Functions and Graphs

Worked example: a taxi charges a fixed 500 plus 150 per kilometre: cost C = 150d + 500 for distance d. The intercept 500 is the charge at zero distance — the flag fall. The slope 150 is the rate, cost per kilometre. A 12 km ride: C = 150×12 + 500 = 2,300. Formula, graph and story are the same fact three ways.

From Chapter 5 — Introducing Calculus — Rates and Totals

Worked example: $y = -x^2 + 6x$ models yield against fertiliser units. $dy/dx = -2x + 6$; setting it to zero gives $x = 3$ — the peak. Check: $y(3) = -9 + 18 = 9$, against $y(2) = 8$ and $y(4) = 8$. The derivative found the optimum the table confirms: three units, no more, because beyond it yield falls.

From Chapter 6 — Mathematical Thinking and Proof

Worked example: claim — the sum of two odd numbers is even. Proof: an odd number is one more than a multiple of two, so write the numbers as $2a + 1$ and $2b + 1$. Their sum is $2a + 2b + 2 = 2(a + b + 1)$, visibly a multiple of two, hence even. No examples were needed; the argument covers every pair of odd numbers that will ever exist.

Further Practice — Review and Discussion

Work through these prompts in writing; they are the revision spine of this book. Answer in full sentences, with examples from your own place of study wherever the prompt allows.

Chapter 1 — Numbers, Powers and Roots

- Explain in your own words, with one example: Fraction, decimal and percentage are one fact in three clothes.
- Explain in your own words, with one example: Index laws are bookkeeping for repeated multiplication.
- Explain in your own words, with one example: Estimate first; compute second; compare always.

Discussion: where does the subject of this chapter — numbers, powers and roots — touch your own community, farm, school or workplace? Write one page, naming real places and real constraints, and end with one question you would research next.

Chapter 2 — Algebra — The Language of Relationships

- Explain in your own words, with one example: Collect like terms only.
- Explain in your own words, with one example: Expanding and factorising are one road, two directions.
- Explain in your own words, with one example: Rearrange by undoing operations equally on both sides.

Discussion: where does the subject of this chapter — algebra — the language of relationships — touch your own community, farm, school or workplace? Write one page, naming real places and real constraints, and end with one question you would research next.

Chapter 3 — Equations — Linear and Quadratic

- Explain in your own words, with one example: Name quantities with letters; word problems fall after that.
- Explain in your own words, with one example: Two unknowns require two equations.
- Explain in your own words, with one example: The discriminant foretells two, one or no real solutions.

Discussion: where does the subject of this chapter — equations — linear and quadratic — touch your own community, farm, school or workplace? Write one page, naming real places and real constraints, and end with one question you would research next.

Chapter 4 — Functions and Graphs

- Explain in your own words, with one example: A function gives exactly one output per input.
- Explain in your own words, with one example: In y = mx + c, m is rate of change and c is the starting value.
- Explain in your own words, with one example: Read graphs for intercepts, turning points and trend before detail.

Discussion: where does the subject of this chapter — functions and graphs — touch your own community, farm,

school or workplace? Write one page, naming real places and real constraints, and end with one question you would research next.

Chapter 5 — Introducing Calculus — Rates and Totals

- Explain in your own words, with one example: The derivative is the slope at a point; for x^n it is nx^{n-1}.
- Explain in your own words, with one example: Integration accumulates totals and reverses differentiation.
- Explain in your own words, with one example: Rates and totals are two views of one process.

Discussion: where does the subject of this chapter — introducing calculus — rates and totals — touch your own community, farm, school or workplace? Write one page, naming real places and real constraints, and end with one question you would research next.

Chapter 6 — Mathematical Thinking and Proof

- Explain in your own words, with one example: Examples suggest; only proof establishes.
- Explain in your own words, with one example: One counterexample refutes any conjecture.
- Explain in your own words, with one example: Define your letters; order your steps; conclude plainly.

Discussion: where does the subject of this chapter — mathematical thinking and proof — touch your own community, farm, school or workplace? Write one page,

naming real places and real constraints, and end with one question you would research next.

Glossary of Key Terms

Index (power) — The number of times a base is multiplied by itself.

Expression — A combination of numbers, letters and operations.

Factorise — To write an expression as a product of factors.

Quadratic — An equation or expression involving x^2 as its highest power.

Discriminant — $b^2 - 4ac$, which foretells a quadratic's real solutions.

Function — A rule giving exactly one output for each input.

Derivative — The rate of change of a function at a point; the slope of its graph there.

Integral — The accumulation of a quantity; the reverse of differentiation.

End of book. Nji Griphan · griphan.48hourrfprescue.com · info@griphan.48hourrfprescue.com